**臺北巿立大學**

**103學年度第一學期學士班二、三年級轉學生招生考試試題**

**系 別：**數學系（三年級）

不得使用計算機或任何儀具。

**科 目：**高等微積分

**考試時間：**90分鐘【8:20−9:50】

**總 分：**100分

* **注意：**不必抄題，作答時請將試題題號及答案依照順序寫在答卷上；**限用藍色或黑色筆作答**，使用其他顏色或鉛筆作答者，所考科目以零分計算。**(於本試題紙上作答者，不予計分。)**

**計算證明題（每題10分，共100分）**

1.Prove that every bounded sequence in $R$ has a subsequence that converges to some point in $R$.

2.Let  be a sequence in $R$, and ** for all$ n\in N$. Show that ****.

3.Let$ f:[a,b]\rightarrow R$ be a continuous function on a closed bounded interval . Prove that  is integrable on .

4.Suppose that$ f:R^{n}\rightarrow R $is a function, and each of the partials $\frac{∂f}{∂x\_{i}}$ exists and is continuous on$ R^{n}$. Prove that *f* is differentiable on $ R^{n}$.

5.Determine the third-order Taylor formula for the function  about the point $(1,\frac{π}{2})$.

6.Let $x\_{n}$ be a sequence of real numbers such that$ \left|x\_{n}-x\_{n+1}\right|\leq \frac{1}{2^{n}}$.

Show that $x\_{n}$ converges.

7.Prove that the series $\sum\_{k=1}^{\infty }\left( \frac{x^{k}}{k!} \right)^{2} $converges uniformly on$ R.$

8.Prove that the unit interval (0, 1) in $R$ is uncountable.

9. Let *M* be a metric space and a set $A⊂M$. *A* is closed if and only if the accumulation points of *A* belong to *A*.

10. Let $A⊂R^{n}$ be open and $B⊂R^{n}$. Define

 $A+B=\{x+y\in R^{n}|x\in A, y\in B\}$.
 Prove $A+B$ is open.