臺灣教育大學系統

104 學年度第一學期學士班二、三年級轉學生聯合招生考試試題

系　　別：應數組
科　　目：微積分
考試時間：90 分鐘【8:30−10:00】
總　　分：100 分

不得使用計算機或任何儀具。

※　**注意：**不必抄題，作答時請將試題題號及答案依照順序寫在答卷上；**限用藍色或黑色筆作答**，使用其他顏色或鉛筆作答者，所考科目以零分計算。**(於本試題紙上作答者，不予計分。)**

計算證明題（每題 10 分，共 100 分）

1. Prove that if f is continuous on $[0,1]$ and satisfies $0 \le f(x) \le 1$ for all $x \in [0,1]$, then there exists at least one point c in $[0,1]$ such that $f(c) = c$.

2. Assume that y is a twice differentiable function of x which satisfies the equation $y^2 + xy - x^2 = 9$. Express $\frac{d^2y}{dx^2}$ in terms of x and y.

3. Find a and b given that $f(x) = \frac{ax}{(x^2 + b^2)}$ has a local minimum at $x = -2$ and $f'(0) = 1$.

4. Evaluate, if possible, the improper integral $\int_0^\infty \frac{e^x}{e^{2x} + 1} dx$.

5. Evaluate $\iint_S \frac{1}{x^2 + y^2} dxdy$, where S is the region between the circles $x^2 + y^2 = 4$ and $x^2 + y^2 = 9$.

6. Suppose that a function f is differentiable on the open interval (a,b) and continuous on the closed interval $[a,b]$. Prove that there is at least one number c in (a,b) such that $f(b)-f(a)=f'(c)(b-a)$.

7. Calculate the limit $\lim_{x\to 0^+}(\sin x)^x$.

8. Find equations for the lines tangent and normal to the curve

$$y\sin 2x - x\sin y = \pi/4$$

at the point $(\pi/4, \pi/2)$.

9. Calculate the area of the region bounded by the curves

$$x^2 - 2xy + y^2 + x + y = 0, \qquad x + y + 4 = 0.$$

10. Find the interval of convergence of the power series

$$\frac{1}{5^2}(x-1)+\frac{4}{5^4}(x-1)^2+\frac{9}{5^6}(x-1)^3+\frac{16}{5^8}(x-1)^4+\cdots$$