

臺北市立大學

105 學年度第一學期學士班二、三年級轉學生招生考試試題

系 別：數學系（二年級）

科 目：微積分

考試時間：90 分鐘【8:30–10:00】

總 分：100 分

不得使用計算機
或任何儀具。

※ 注意：不必抄題，作答時請將試題題號及答案依照順序寫在答卷上；限用藍色或黑色筆作答，使用其他顏色或鉛筆作答者，所考科目以零分計算。（於本試題紙上作答者，不予計分。）

計算證明題（每題 10 分，共 100 分）

1. Find $\lim_{x \rightarrow 1} f(x)$; $f(x) = \begin{cases} 2x, & x < 1, \\ x^2 + 1, & x > 1. \end{cases}$
2. Calculate the integral $\int_0^2 \int_x^2 x\sqrt{1+y^3} dy dx$.
3. Let $f(x) = \frac{1}{4}x^3 + x - 1$
 - (a) What is the value of $f^{-1}(x)$ when $x = 3$?
 - (b) What is the value of $(f^{-1})'(x)$ when $x = 3$?
4. Find the area of the surface obtained by rotating the curve $y = x^2, 0 \leq x \leq 1$ about the y -axis. (寫出積分算式即可)
5. Use the Lagrange Multipliers to find the minimum value of $f(x, y, z) = 2x^2 + y^2 + 3z^2$ subject to the constraint $2x - 3y + 4z = 49$.
6. Find the points on the curve $(x^2 + y^2)^2 = x^2 - y^2$ at which the tangent line is horizontal.
7. Set $f(x) = x^2(x-7)^{1/3}$. (a) Find the intervals on which f increases and the intervals on which f decreases. (b) Determine whether the graph of f has any vertical tangents or vertical cusps.
8. Find $H'(\pi/4)$ given that $H(x) = \int_{\tan x}^{2x} t\sqrt{1+t^2} dt$.
9. Find the interval of convergence of the power series $\frac{1}{16}(x+1) - \frac{2}{25}(x+1)^2 + \frac{3}{36}(x+1)^3 - \frac{4}{49}(x+1)^4 + \dots$
10. Find the volume of the solid bounded above by the cone $z^2 = x^2 + y^2$ and below by the region which lies inside the curve $x^2 + y^2 = 3x$.