臺北市立大學

107 學年度第一學期學士班二、三年級轉學生招生考試試題

系　　別：數學系（二年級）
科　　目：微積分
考試時間：90 分鐘【8:30－10:00】
總　　分：100 分

> 不得使用計算機
> 或任何儀具。

※　注意：不必抄題，作答時請將試題題號及答案依照順序寫在答卷上；**限用藍色或黑色筆作答**，使用其他顏色或鉛筆作答者，所考科目以零分計算。(**於本試題紙上作答者，不予計分。**)

計算證明題（每題 10 分，共 100 分）

1. If y satisfies $\dfrac{dy}{dt} = ky^2(M - y^2)$, where k、M are constants, it proves that y will also satisfy $\dfrac{d^2y}{dt^2} = 2k^2y^3(M - y^2)(M - 2y^2)$.

2. A cubic polynomial $f(x) = ax^3 + bx^2 + cx + d$ has two relative extrema on the interval $I = [-3, 3]$, which are $f(2) = 3$ and $f(-1) = 0$, respectively. What is this polynomial?

3. What is the value of $\displaystyle\lim_{h \to 0} \dfrac{\sqrt{5 + 2h} - \sqrt{5 - 2h}}{h}$?

4. Suppose f is a function that satisfies the equation

   $f(x + y) = f(x) + f(y) + 3xy^2 + 3x^2y$ for all real numbers x and y.

   Suppose also that $\displaystyle\lim_{x \to 0} \dfrac{f(x)}{x} = 1$. Find $f'(x)$.

5. Evaluate $\displaystyle\int \dfrac{x^5}{\sqrt{1 - 2x^3}}dx$.

6. Give an $\varepsilon - \delta$ proof for the statement $\displaystyle\lim_{x \to 1/2} x^5 = 1/32$.

7. Find equations for the tangent and normal lines to the curve $\tan xy = x$ at the point $(1, \pi/4)$.

8. Calculate the integral $\int \dfrac{1}{\sqrt{1+e^x}}\,dx$.

9. Find the interval of convergence of the series $\displaystyle\sum_{k=1}^{\infty} \dfrac{(-1)^k}{k^2 3^k}(x+2)^k$.

10. Let Ω be the first-quadrant region bounded by the curves $xy=1$, $xy=4$, $y=x$, $y=4x$, Determine the area of Ω.