臺北市立師範學院

九十四學年度研究所碩士班入學考試試題

所　　別：數學資訊教育研究所數學教育組

科　　目：基礎數學

考試時間：九十分鐘

總　　分：一百分

※　注意：不必抄題，作答時請將試題題號及答案依照順序寫在答卷上。**(<u>於本試題紙上作答者，不予計分</u>)**

一、微積分試題(50%)

1. If the function $f : \mathrm{R} \to \mathrm{R}$ is defined by

$$f(x) = \begin{cases} 1 & \text{if } x \in \mathrm{Q} \\ 0 & \text{if } x \notin \mathrm{Q} \end{cases}$$

Prove that f is everywhere discontinuous. (10%)

2. Let $f(x) = \begin{cases} x^2 \sin(1/x) & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$. Show that f is differentiable at 0 and give $f'(0)$. (10%)

3. Find $\int \dfrac{x^3 - 8}{x^2 (x-1)^3}\, dx.$ (10%)

4. The region Ω between $y=\sqrt{x}$ and $y=x^2$, $0\le x\le 1$, is revolved around the line $x=-2$.
Find the volume of the solid which is generated. (10%)

5. Let T be the solid bounded below by the half-cone $z=\sqrt{x^2+y^2}$ and above by the spherical surface $x^2+y^2+z^2=1$.

Evaluate $\iiint\limits_T e^{(x^2+y^2+z^2)^{3/2}}\,dxdydz.$ (10%)

二、線性代數(50%)

1. Let $A_n=\begin{pmatrix} 1 & 1 & 1 & 1 & \dots & 1 \\ 1 & 2 & 2 & 2 & \cdots & 2 \\ 1 & 2 & 3 & 3 & \cdots & 3 \\ 1 & 2 & 3 & 4 & \cdots & 4 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & 2 & 3 & 4 & \cdots & n \end{pmatrix}$. Prove that $\det(A_n)=1$, for all positive integer n. (10%)

2. Let I_n be an $n\times n$ identity matrix.

Can you find a symmetric matrix A and an antisymmetric matrix B such that $AB=I_n$? Explain your reasons. (10%)

3. Let A be a diagonalizable 2×2 matrix over R, I be the 2×2 identity matrix, and **0** be the 2×2 zero matrix. The matrix A satisfies the equation $A^2-2A-3I=0$.

(a) Find all the possible matrices A and explain your reasons. (15%)

(b) Find all the minimal polynomials of the corresponding matrices A and explain your reasons. (15%)