

臺北市立師範學院九十四學年度 學士班二年級轉學生入學考試試題

系 別：數學資訊教育學系數學組

科 目：微積分

考試時間：九十分鐘

總 分：100 分

※注意：不必抄題，作答時請將試題題號及答案依照順序寫在答卷上。(於本試題紙上作答者，不予計分)

※說明：每題 10 分，共 100 分。

1. Let $f(x) = x^2 \sin\left(\frac{1}{x}\right)$ for $x \neq 0$ and $f(0) = 0$. Find $f'(0)$.

2. Determine whether series $\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^3}$ is convergent or divergent.

3. Let $\{a_n\}$ be a sequence defined by $a_1 = 2, a_{n+1} = \frac{1}{2}\left(a_n + \frac{2}{a_n}\right)$. Determine whether

$\{a_n\}$ converges or diverges. If it converges, find $\lim_{n \rightarrow \infty} a_n$.

4. $1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \frac{1}{6} + \frac{1}{7} - \frac{1}{8} + \frac{1}{9} - \frac{1}{10} + \frac{1}{11} - \frac{1}{12} + \dots = ?$

5. Find $\int \frac{x+19}{(2x+1)(x^2+9)} dx$.

6. Let $F(x) = \int_{x^2}^1 (t - \sin^2 t) dt$. Find the second derivative $F''(0)$.

7. Evaluate the integral $\int_0^2 \int_x^2 x \sqrt{1+y^3} dy dx$.

8. Find the volume of the ellipsoid $\frac{x^2}{4} + \frac{y^2}{4} + \frac{z^2}{3} = 1$.

(第一頁，共兩頁)

9. Prove that the integral $\int_{-\infty}^x e^{-t^2/2} dt$ converges for all real x .

10. Let u be a continuous function on the closed interval $[a, b]$. Prove the mean value theorem for

definite integrals for u : There exists a number $c \in [a, b]$ such that $u(c) = \frac{1}{b-a} \int_a^b u(t) dt$.